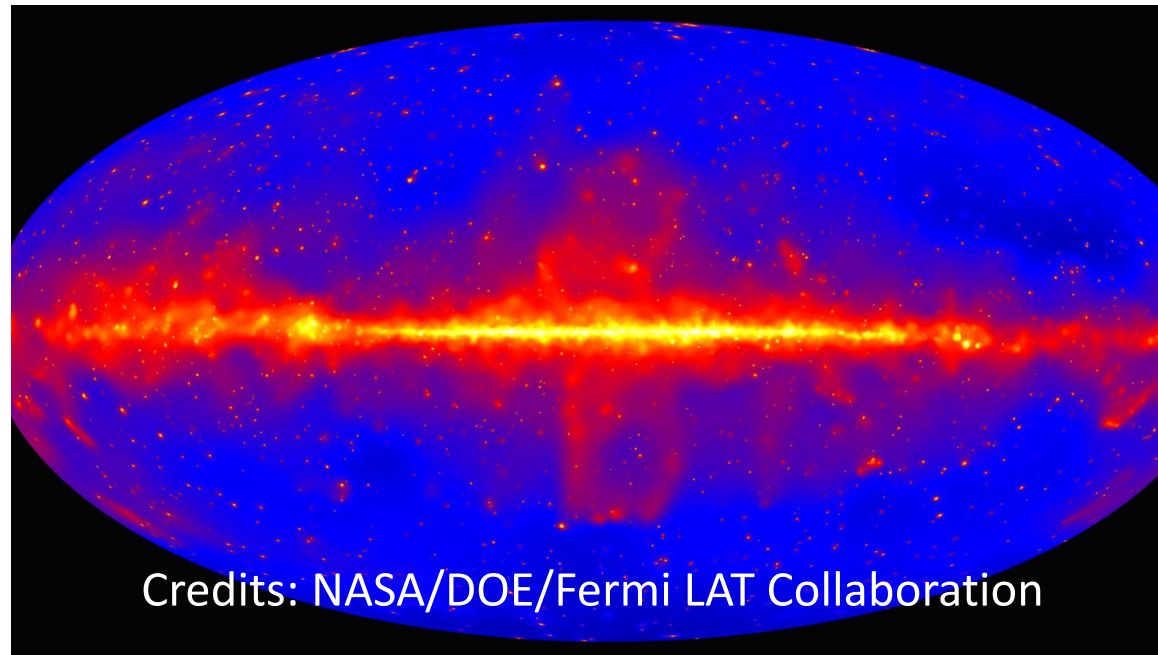


A SHORT HISTORY OF THE PHOTON

Fulvio Parmigiani



"1951 **Einstein** in a letter to Michele Besso, 12 Dec. 1951 :**“All those 50 years of careful pondering have not brought me closer to the answer to the question: ‘What are light quanta?’ Today any old scamp believes he knows, but he’s deluding himself.”**

A CONSERVATIVE REVOLUTIONARY

THE DAWN OF QUANTUM MECHANICS

Planck heard about this conflict between the two fit formulas when Heinrich Rubens was visiting him at his home in the Grunewald suburb of Berlin on 7 October 1900. A few hours later he was able to produce an interpolation formula, which approaches the Rayleigh-Jeans limit for lower frequencies ν and approaches the Wien limit for high ν , with a smooth transition in between.

Planck was just trying to construe in his second quantum theory (see above)—to regard this quantization as merely an epiphenomenon of the interaction between radiation and matter: perhaps we may be allowed to assume that an oscillating resonator does not have a continuously variable energy, but that its energy is a simple multiple of an elementary quantum instead. I believe that by using this theory one can arrive at a satisfactory theory of radiation.

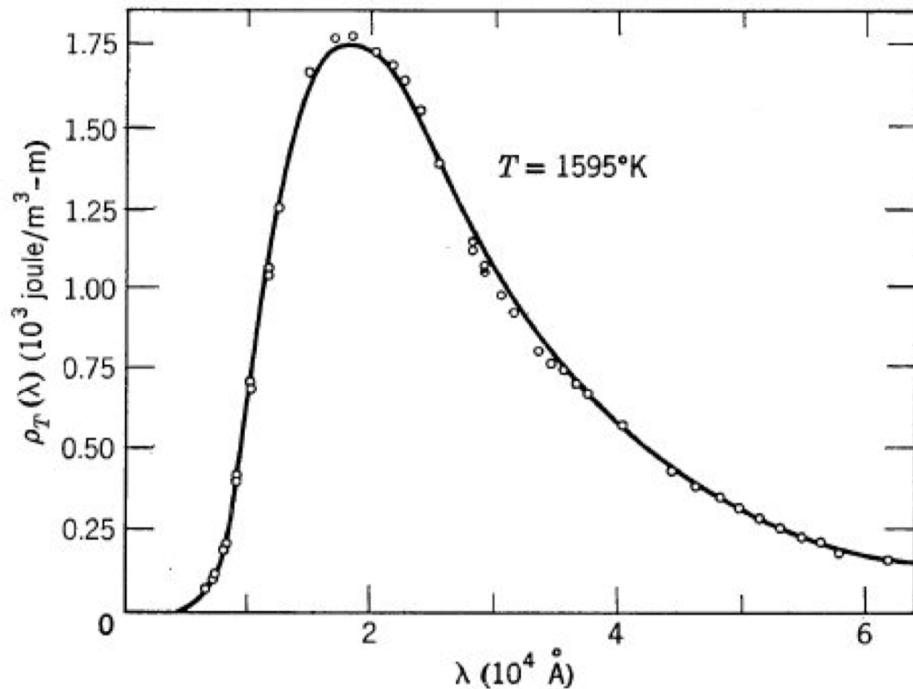
In this formula k_B is the Boltzmann constant of statistical mechanics and h is the quantum of action that Planck had already introduced into the discussion in 1899 and was later named after him. The examinations by Rubens and Kurlbaum in the long-wave range, and by Lummer and Pringsheim in the short-wave range demonstrated a surprisingly good empirical match. *[For the experimental research on black-body radiation conducted around 1900, see Lummer and Pringsheim (1897–1900), Kurlbaum and Lummer (1898, 1901), Rubens and Nichols (1896), Rubens (1917), Rubens and Kurlbaum (1900, 1901); further Kangro (1970, 1970/71)*



Max Planck,
(1858-1947)

$$\rho(\nu, T) = \frac{8\pi\nu^2}{c^3} \frac{h\nu}{e^{h\nu/k_B T} - 1}$$

NOT EASY TO ACCEPT



In the Planck's equation k_B is the **Boltzmann** constant of statistical mechanics and h is the quantum of action that Planck had already introduced into the discussion in 1899 and was later named after him.

The examinations by Rubens and Kurlbaum in the long-wave range, and by Lummer and Pringsheim in the short-wave range demonstrated a surprisingly good empirical match.

Physically, Planck's dramatic explanation of blackbody radiation includes three fundamentally new ideas:

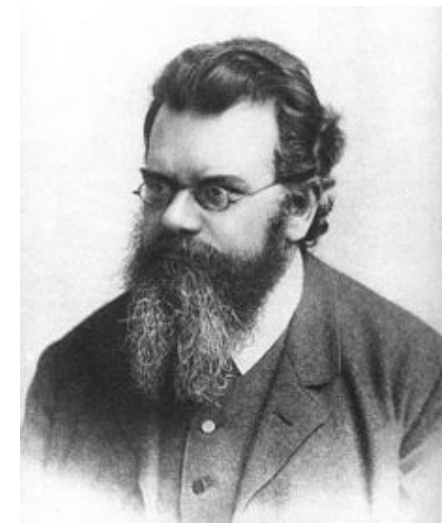
- 1-The energy of a system can take only discrete values, which are represented on its energy-level diagram.**
- 2-A quantized oscillator can gain or lose energy only in discrete amounts, which are related to its frequency by $\Delta E = h\nu$.**
- 3. To emit energy from higher energy states, the temperature of a quantized system must be sufficiently high to excite those states.**

These three ideas have permeated all areas of science and technology. They are the basis for our understanding that energy (like matter) is discrete, not continuous, and that it can be transferred only in discrete chunks and not by arbitrary amounts.

PLANCK AND BOLTZMANN

In order to be able to apply this method, Planck had to divide the energy up into finite packets, to be able to perform the combinatorics in the manner of Boltzmann.

How dire this situation must have been for Planck to make him venture this formal step toward quantized energy, is revealed in his own words in a letter to the American experimental physicist **Robert Williams Wood (1868–1958)** from 1931:



Ludwig E. Boltzmann
1844 -1906

“In a word, I could call the whole deed an act of desperation. For I am, by nature, peaceable and not inclined to dubious adventures. But I had been wrestling with the problem of the equilibrium between radiation and matter for 6 years [since 1894], without success. I knew that this problem was of fundamental importance to physics. I was familiar with the formula describing the energy distribution in a normal spectrum; consequently, a theoretical interpretation had to be found at any price, no matter how high. Classical physics was not good enough, that was clear to me. Because, according to it, over time the energy in matter must convert entirely into radiation. In order for it not to do so, we need a new constant [Planck’s quantum of action h] that assures that the energy not disintegrate. [...] one finds that this dissipation of energy as radiation can be prevented by the assumption that energy be compelled from the outset to stay together in specific quanta. That was a purely formal assumption and I did not really consider it much, just that I must, under all conditions, cost what it may, force a positive result.”

PLANCK AND BOLTZMANN

Planck started an intense search for a satisfactory and reasonable way to derive the formula from more general considerations. **In December 1900 he finally succeeded, but it came at a high price. He used a statistical method by Boltzmann that he and his assistant Zermelo had already heftily criticized, to calculate the entropy S from the number of macroscopically indistinguishable microscopic ‘complexions’ K , that is, from distributions of the total available energy onto the individual resonators.**

Planck was trying to avoid any outright quarrel with the solid foundations of classical physics.

It seems to have been downright embarrassing for him that with his initially harmless interpolation proposal in 1900 to explain the distribution of radiative energy, of all people would have triggered such a far-reaching development. Consequently, Planck did everything he could, particularly with his so-called second quantum theory between 1907 and 1911, to mend the rupture again.

In 1986 John Heilbron very fittingly characterized Planck as a “conservative revolutionary”—one could also say: a renitent revolutionary, because he neither sought nor wanted to assume this role as discoverer of energy quantization, as has often been ascribed to him in the later historiography

PLANCK AND EINSTEIN

Irrespective of the reading one might choose for Planck's writings from around 1900, Einstein's approach was definite. Quite in contrast to Planck, he offensively set out in search of cracks, pointed them out and took them as his starting point toward something new. Evidently, the no longer extant first draft of his paper, in which he criticized Planck's half-hearted position on energy quantization, was much more aggressively direct. But Einstein's close friend and colleague at the Bernese patent office, Michele Besso (1873–1955) persuaded him to soften the tone a bit and present his hypotheses about the light quantum much more deferentially

Einstein characterized Planck's way of going about this in a later talk:

To solve the problem of radiation, Planck concluded that one would have to introduce a new physical quantity: the famous quantity h , in order to arrive at a reasonable formula for radiation. But this calculational figure has a very real meaning in nature, in the sense that radiation forms or vanishes only in the magnitude $h\nu$. When a bell rings, it sounds loudly when it is struck firmly, and more quietly, the weaker it is struck; it receives a greater or lesser amount of energy. In radiative processes, this is not so to the same degree; rather, energy cannot be introduced into a luminous structure in arbitrarily small amounts, never less than one quantum and always only integral multiples of this quantum are taken up or released again by a structure able to radiate

PLANCK AND EINSTEIN

1913: Einstein is formally proposed for Membership in the Prussian Academy. To ensure Einstein's election to the Prussian Academy, the four most important German physicists and members of the Academy, Planck, Nernst, Rubens and Warburg submitted a request to the Prussian Ministry of Education.²⁸ They first announced in the physical-mathematical class of the Prussian Academy of Sciences that they would submit a proposal for membership at the next session. The identity of the candidate was not given.

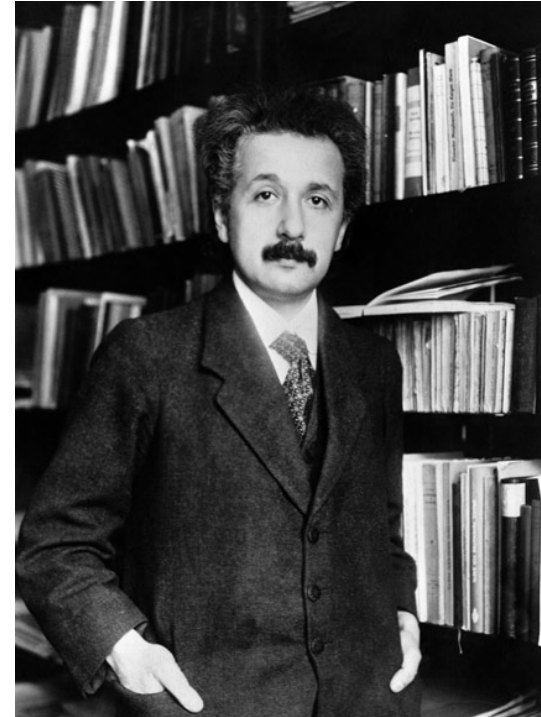
Two weeks later they proposed Einstein for election as a regular member of the Academy. Planck summarized that it can be said that among the major problems, with which modern physics is so rich, there is no one in which Einstein did not take a position in a remarkable manner.

But added Planck, *"That he might have in his speculations, occasionally, overshot the target, as for example in his light quantum hypothesis, should not be counted against him too much; because without taking a risk, even in the most exact science, one is not driven to real innovation"*.

EINSTEIN AND THE QUANTA OF LIGHT

Einstein was very interested in the photoelectric effect and its interpretation and kept abreast of the debates about the latest experimental data. In 1905 he decided— after much deep thought—to reconsider the experiment using a completely different kind of model.

«...our prevailing [conventional electromagnetic] conceptions cannot explain why radiation of higher frequency should produce elementary processes of greater energy than radiation of lower frequency. In brief, we neither understand the specific effect of frequency nor the lack of a specific effect of intensity.»



Thus a double failure of the classical theories confronted him. First, they contrafactually presumed a proportionality of 1 : 1000 for the intensity, which was definitely not experimentally establishable, not even for variations in intensity. Second, they were not able to explain this “specific effect of frequency” of the radiation either.

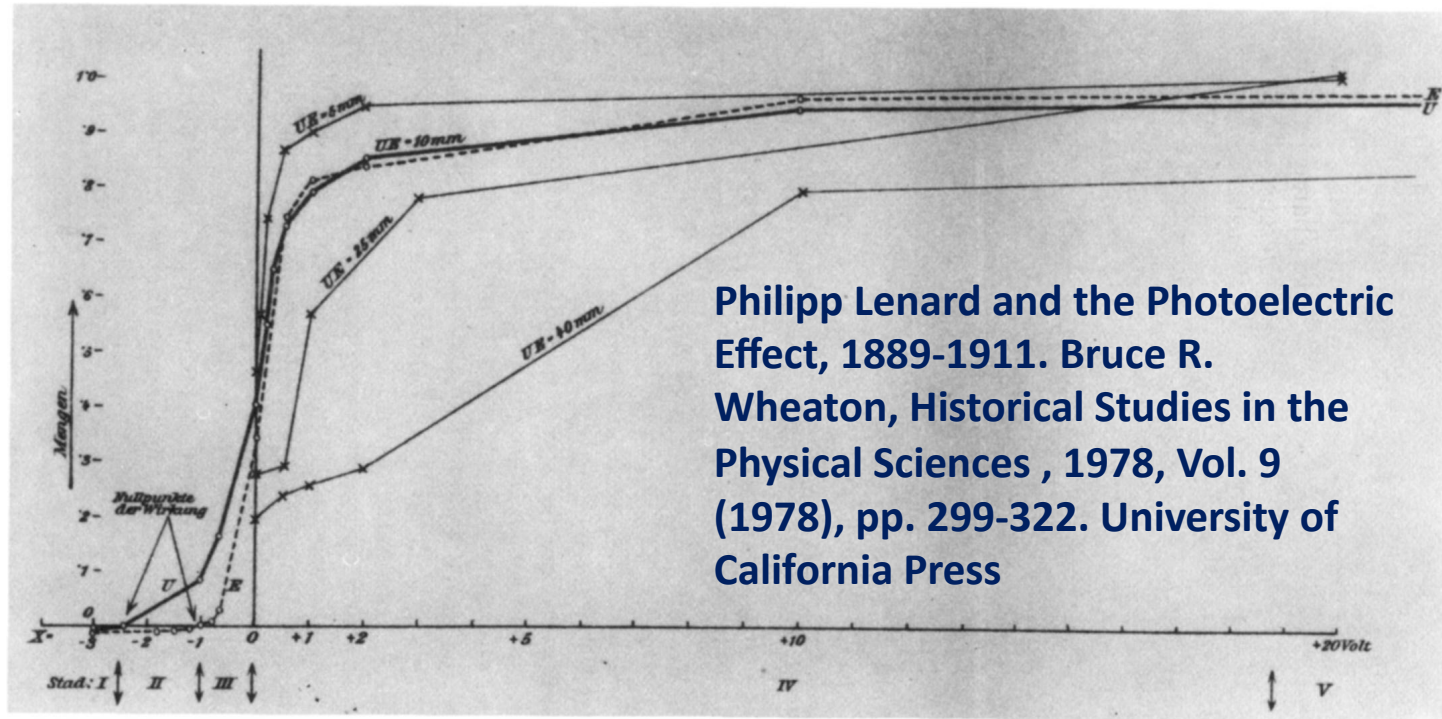
THE PHOTOELECTRIC EFFECT

THE EARLY DAYS OF THE PHOTOELECTRIC EFFECT

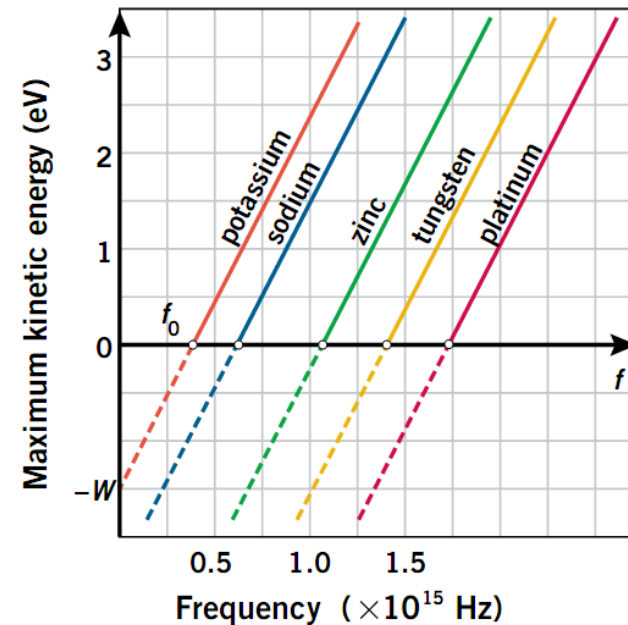
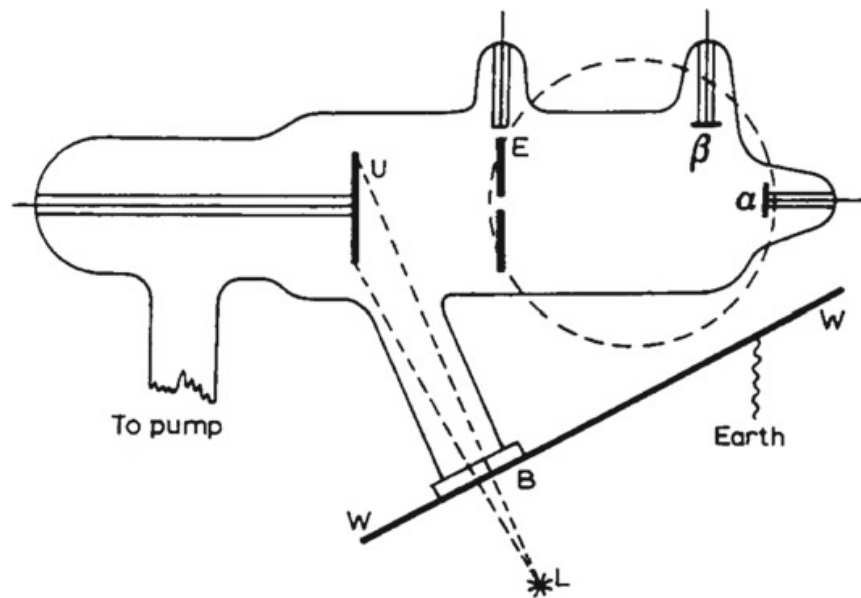


Hertz, Heinrich (1887). "*Ueber den Einfluss des ultravioletten Lichtes auf die elektrische Entladung*". *Annalen der Physik* **267**: S. 983–1000.

Lenard, Peter (1902). "*Ueber die lichtelektrische Wirkung*". *Annalen der Physik*, **313**: 149–198.



EINSTEIN AND THE QUANTA OF LIGHT



Lenard's cathode-ray tube from 1902 with a UV-permeable aluminium window B, cathode U, anode α and a negatively charged grid E that permits the stream of photons triggered by the UV radiation to be regulated. *Source* The Nobel lecture by Philipp Lenard (1906b) p. 122.

In 1902, Lenard observed that the energy of individual emitted electrons increased with the frequency (which is related to the color) of the light. This appeared to be at odds with Maxwell's wave theory of light, which predicted that the electron energy would be proportional to the intensity of the radiation.

Lenard observed the variation in electron energy with light frequency using a powerful electric arc lamp which enabled him to investigate large changes in intensity, and that had sufficient power to enable him to investigate the variation of the electrode's potential with light frequency. He found the electron energy by relating it to the maximum stopping potential (voltage) in a phototube. He found that the maximum electron kinetic energy is determined by the frequency of the light.

THE EARLY DAYS OF THE PHOTOELECTRIC EFFECT



A. Einstein
Nobel prize 1921

Theoretical explanation by A. Einstein (1905):
QUANTIZATION OF LIGHT ←

ANSATZ

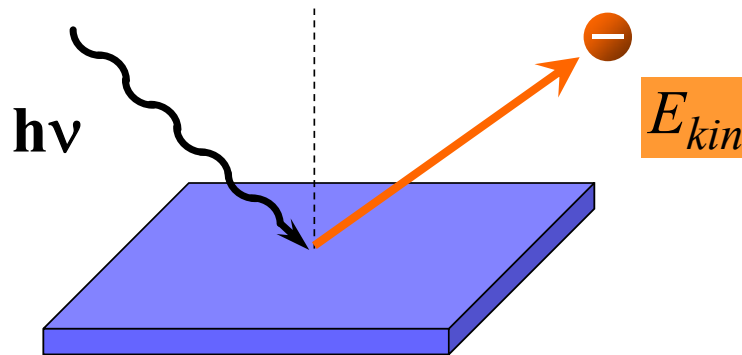
Ann. d. Phys. 17, 132 (1905):
Die kinetische Energie solcher Elektronen ist

$$\frac{R}{N} \beta \nu - P.$$

$$E_{kin}^{max} = h \nu - \Phi$$

Planck's
constant

photocathode
workfunction



In 1905, [Einstein](#) proposed a theory of the photoelectric effect assuming that light consists of light quanta. Each packet carries energy that is proportional to the frequency ν of the corresponding electromagnetic wave. The proportionality constant was the [Planck constant](#). This was measured by Millikan.

EINSTEIN AND THE QUANTA OF LIGHT

The wave theory of light, which operates with continuous spatial functions, has worked well in the representation of purely optical phenomena and will probably never be replaced by another theory. It should be kept in mind, however, that the optical observations refer to time averages rather than instantaneous values. In spite of the complete experimental confirmation of the theory as applied to diffraction, reflection, refraction, dispersion, etc., it is still conceivable that the theory of light which operates with continuous spatial functions may lead to contradictions with experience when it is applied to the phenomena of emission and transformation of light.

It seems to me that the observations associated with blackbody radiation, fluorescence, the production of cathode rays by ultraviolet light, and other related phenomena connected with the emission or transformation of light are more readily understood if one assumes that the energy of light is discontinuously distributed in space. In accordance with the assumption to be considered here, the energy of a light ray spreading out from a point source is not continuously distributed over an increasing space but consists of a finite number of energy quanta which are localized at points in space, which move without dividing, and which can only be produced and absorbed as complete units.



The Nobel Prize in Physics 1921 was awarded to Albert Einstein "*for his services to Theoretical Physics, and especially for his discovery of the law of the photoelectric effect*".

AMERICAN JOURNAL of PHYSICS

A Journal Devoted to the Instructional and Cultural Aspects of Physical Science

VOLUME 33, NUMBER 5

MAY 1965

Einstein's Proposal of the Photon Concept—a Translation of the
Annalen der Physik Paper of 1905*

A. B. ARONS† and M. B. PEPPARD‡
Amherst College, Amherst, Massachusetts

A VITAL QUESTION

A LONG LIVING CONTROVERSY

1911, **Arnold Sommerfeld** (Society of German Scientists and Physicians) :

«*The theory of energy quanta is an entirely different and problematic current issue [...] The fundamental concepts here are still in a state of flux and the problems are countless. [...] Einstein drew the farthest-reaching consequences of Planck's discovery [...] without, as I believe, maintaining his bold standpoint of the time anymore now.*»

1913 **Robert A. Millikan**, (American Association for the Advancement of Science in Cleveland, Ohio)

«*Lorentz will have nothing to do with any ether-string theory, or spotted wave-front theory, or electro-magnetic corpuscle theory. Planck has unqualifiedly declared against it, and Einstein gave it up, I believe, some two years ago. [...] In conclusion then we have at present no quantum theory which has thus far been shown to be self-consistent, or consistent with even the most important of the facts at hand.*»

1916 Einstein, (Zur Quantentheorie der Strahlung. *Physikalische Zeitschrift* 18: 121–128 (1916): «*the simplicity of the hypotheses, the generality with which the analysis can be carried out so effortlessly, and the natural connection to Planck's linear oscillator (as a limiting case of classical electrodynamics and mechanics).*»

A FRIENDSHIP FOR LIFE: ALBERT AND MICHELE

1917 **Einstein** [letter to **Michele Besso (1873–1955)**] with reference to the same paper:
“**The quantum paper I sent out has led me back to the view of the spatially quantumlike nature of radiation energy. But I have the feeling that the actual crux of the problem posed to us by the eternal enigma-giver is not yet understood absolutely. Shall we live to see the redeeming idea? “**

1951 **Einstein** in a letter to **Michele Besso**, 12 Dec. 1951 :

“**All those 50 years of careful pondering have not brought me closer to the answer to the question: ‘What are light quanta?’ Today any old scamp believes he knows, but he’s deluding himself.”**

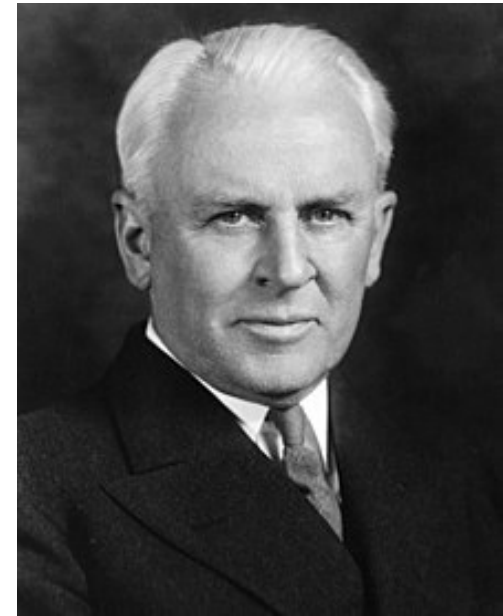
1896 At Zurich Polytechnic, Michele is taking his physics class instructed by [Professor Weber](#). He watches as classmate [Mileva Marić](#) questions Weber on the properties of energy and thermodynamics, and other classmate [Albert Einstein](#) agrees with her, even going on to create an example with Michele's pencils. He later watches as Albert is quieted by Weber, who informs him that theatrics have no place at Zurich Polytechnic.

Later outside the University, Michele and his friend [Marcel Grossmann](#) approach Albert and tell him that they were impressed by his demonstration in class. The two invite Albert to lunch, and a friendship emerges.



MILLIKAN'S CHANGING OPINIONS OF HIS EXPERIMENT

“It was in 1905 that Einstein made the first coupling of photo effects and with any form of quantum theory by bringing forward the bold, not to say reckless, hypothesis of an electro-magnetic light corpuscle of energy $h\nu$, which energy was transferred upon absorption to an electron. This hypothesis may well be called reckless first because an electromagnetic disturbance which remains localized in space seems a violation of the very conception of an electromagnetic disturbance, and second because it flies in the face of the thoroughly established facts of interference [Millikan, R.A. 1914. A Direct Determination of “ h ”. Physical Review 4: 73-75. Millikan, R.A. 1916a. A Direct Photoelectric Determination of Planck’s ‘ h ’. Physical Review 7: 355-388, Millikan, R.A. 1916b. Einstein’s Photoelectric Equation and the Contact Electromotive, Force. Physical Review 7: 18-32]

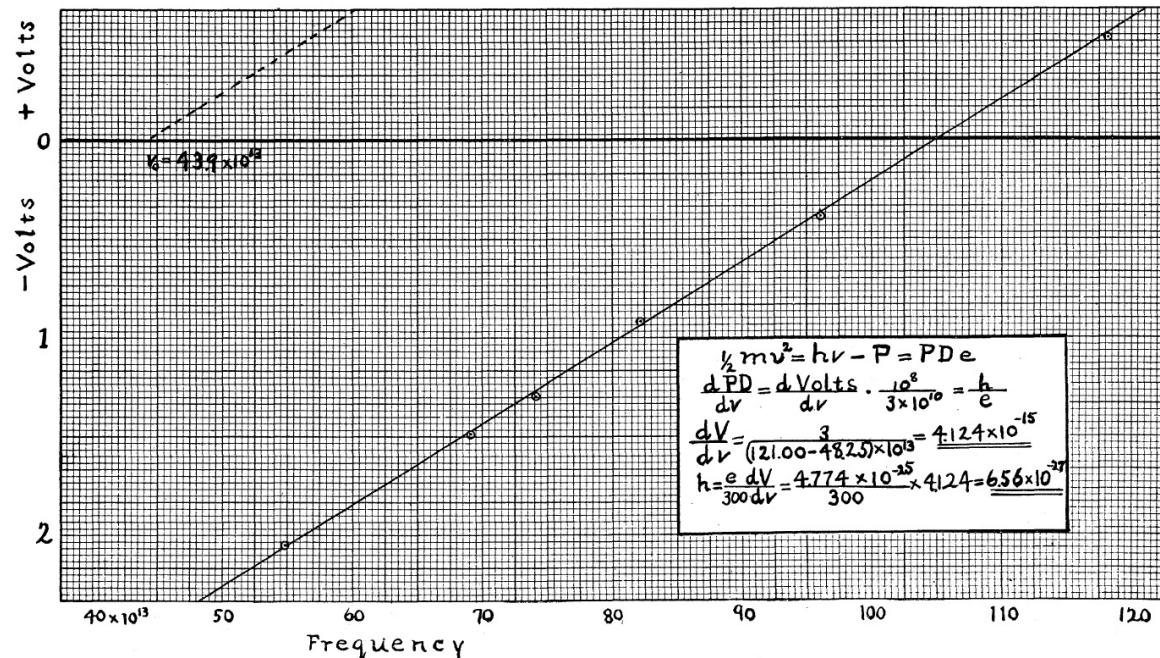


Robert Andrews Millikan
(1868 -1953)

After discussing his confirmation of Einstein’s equation Millikan remarked, **«I thought, that the emitted electron that escapes with the energy $h\nu$ gets that energy by the direct transfer of $h\nu$ units of energy from the light to the electron and scarcely permits any other interpretation than that which Einstein had originally suggested, namely that of the semi-corpuseular or photon theory of light itself.»** [Millikan, R.A. 1950. The Autobiography of Robert A. Millikan. Prentice-Hall, New York.

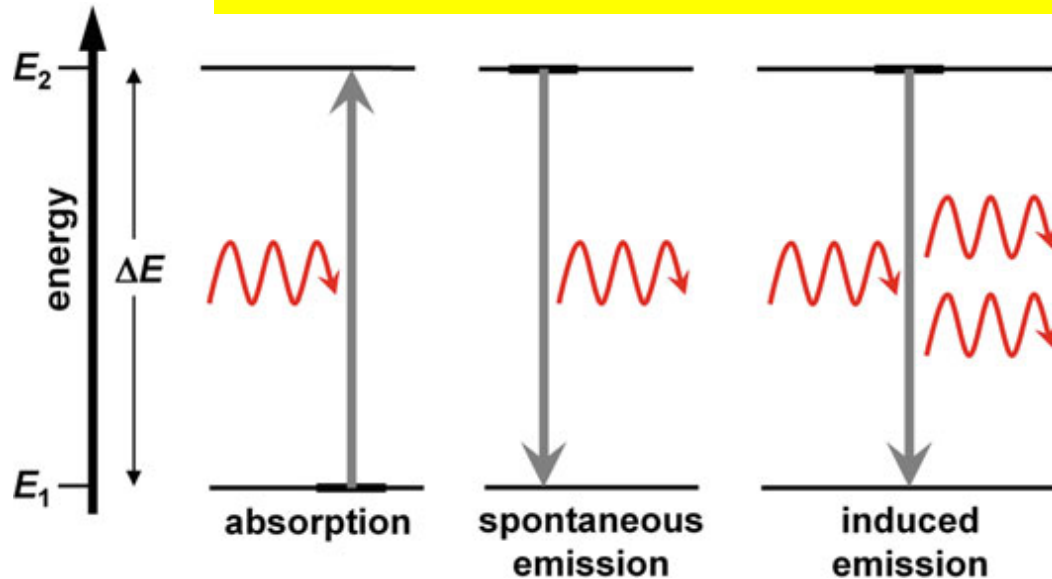
PLANKC'S h CONTANT MEASURED

Millikan's experimental results, 1916: **The constant h , which according to Millikan's plot defines the slope of the interpolation line contained in all his measurements with a precision of $\pm 0.5\%$, came to $6.57 \cdot 10^{-27}$ erg s.** Converted into modern units, it is $6.616 \cdot 10^{-34}$ Js, which is in very good agreement with the modern value for Planck's quantum of action $h = 6.62607 \cdot 10^{-34}$ Js. *Source* Millikan(1916b)



In 1931 Einstein himself remarked, **"I acknowledge gratefully Millikan's researches concerning the photo effect which first proved conclusively that the emission of electrons from solid bodies under the influence of light is associated with a definite period of vibration [frequency] of the light itself, which result of the quantum theory is especially characteristic for the corpuscular structure of radiation [Einstein, A. 1931. Professor Einstein at the California Institute of Technology. Science 73: 375-379]**

AN AMAZINGLY SIMPLE DERIVATION OF PLANCK'S FORMULA



A. Einstein, *Phys. Z.* **18**, 121 (1917). English translation "On the Quantum Theory of Radiation," by D. ter Haar, *The Old Quantum Theory*, Pergamon Press, New York (1967), p. 167

By imposing the conditions for thermal equilibrium in the presence of those processes and invoking the Wien displacement law, Einstein showed that **B_{ba} is proportional to A_{ba}** ; that **$B_{ba} = B_{ab}$** ; that in making a transition, **the atom emits monochromatic radiation with frequency given by the Bohr condition $\nu = (E_b - E_a)/h$** , where h is Planck's constant; and that for the atoms to achieve thermal equilibrium, **$\rho(\nu, T) = (8\pi h\nu^3/c^3)/(\exp[h\nu/kT] - 1)$** . **This last equation is the Planck radiation law.**

Einstein wrote: **"the simplicity of the hypotheses, the generality with which the analysis can be carried out so effortlessly, and the natural connection to Planck's linear oscillator [...] seem to make it highly probable that these are basic traits of a future theoretical representation."** [Einstein, Albert. 1916b. Zur Quantentheorie der Strahlung. *Mitteilungen der Physikalischen Gesellschaft Zürich* 18: 47–62; reprinted in CPAE vol. 6, 1996, doc. 38: 381–398 (trans. ed.: *On the quantum theory of radiation*, 220–233).

SPONTANEOUS AND INDUCED EMISSION: 1916–17

Absorption, stimulated emission and spontaneous emission are evidently introduced by analogy with the behavior of classical oscillators (Abraham-Lorentz oscillator).

Applying the Boltzmann statistics to a two-quantized state system (although it is not obvious why quantized systems should obey classical statistics), Einstein introduced a fundamental character of the **Quantum Physics versus the Classical Physics: the interaction are PROBABILISTIC and NOT DETERMINISTIC.**

Einstein justified his assumption of a spontaneous emission rate by stating, “*The statistical law assumed here corresponds to the law of a radioactive reaction, and the elementary process assumed here corresponds to a relation in which only γ rays are emitted.*”

The “law of a radioactive reaction” is Ernest Rutherford’s law for radioactive decay: $N(t) = N(0) \exp(-t/\tau)$, where τ is a time characteristic of the atom.

At the light of the present days QED it is clear that the spontaneous emission is the only process that require the EM field quantization.

An equally daunting problem for Einstein was the lack of any theory for calculating the spontaneous emission rate. That—and a multitude of other problems on atomic structure and dynamics—would simply have to wait for the creation of a complete quantum theory.

For further reading: Rereading Einstein on Radiation, D. Kleppner; Physics Today 58, 2, 30 (2005); doi: 10.1063/1.1897520

**A CREW OF MEN
WHO CHANGED PHYSICS**

JOHN VON NEUMANN AND PAUL A. M. DIRAC

It seems to be odd to observe that absorption and stimulated emission do not require the EM field to be quantised. After the quantization of the EM field by Paul Dirac in 1927 (Einstein was quite aware of the deficiency, for his 1917 paper where the transition probabilities could not be calculated) and the rise of a more advanced quantum theory based on solid mathematical concept (P. Dirac *Principles of Quantum Mechanics* published by Oxford University Press in 1930) and *Mathematical Foundations of Quantum Mechanics* (1932) by [John von Neumann](#) as an important early work in the development of [quantum theory](#), it was possible to calculate the transition probability from one quantum state to another quantum state. [SEE **APPENDIX 1**]



John von Neumann 1903 -1957)



Paul A. M. Dirac (1902–1984)

THE RISE OF THE QUANTUM THEORY

«Einstein was to be proven right—perhaps more than he later would have liked, though. His modeling of quantum theoretical connections based on transition probabilities between a finite number of quantized initial and final states was used by the young Heisenberg as orientation in conceiving matrix coefficients for his matrix mechanics. Paul A. M. Dirac (1902–1984) then showed in 1927 that spontaneous and induced emission have a place in quantum field theory.

Heisenberg's uncertainty relation was also translated into relativistic quantum mechanics in 1931 and later into quantum electrodynamics as well. Einstein's steadfast opposition to Heisenberg's matrix mechanics, Dirac's operator algebra and other variants of later quantum mechanics with its associated stochastic interpretations is well known. But aren't there a number of allusions to "chance" in Einstein's own paper from 1916? It is remarkable that Einstein enclosed the two instances of this word there in scare quotes. »

[K. Hentschel, Photons the history and mental models of light quanta –Springer (2018)]

«Ironically, when the quantum theory, which was needed to fully realize the vision of his radiation theory, was developed between 1925 and 1928, Einstein turned his back on it. His supernatural intuition finally failed him. » [D. Kleppner, Physics Today 58, 2, 30 (2005); doi: 10.1063/1.1897520]

Zur Quantentheorie der Strahlung.

Von A. Einstein¹⁾.

Die formale Ähnlichkeit der Kurve der chromatischen Verteilung der Temperaturstrahlung mit Maxwell'schen Geschwindigkeits-Verteilungsgesetz ist zu frappant, als daß sie lange hätte verborgen bleiben können. In der Tat wurde bereits W. Wien in der wichtigen theoretischen Arbeit, in welcher er sein Verschiebungsgesetz

$$\rho = \nu^3 f\left(\frac{\nu}{T}\right) \quad (1)$$

ableitete, durch diese Ähnlichkeit auf eine weitergehende Bestimmung der Strahlungsformel geführt. Er fand hierbei bekanntlich die Formel

$$\rho = a \nu^3 e^{-\frac{h\nu}{kT}}, \quad (2)$$

welche als Grenzesetz für große Werte von

The opening paragraph of Einstein's 1917 paper, "On the Quantum Theory of Radiation."

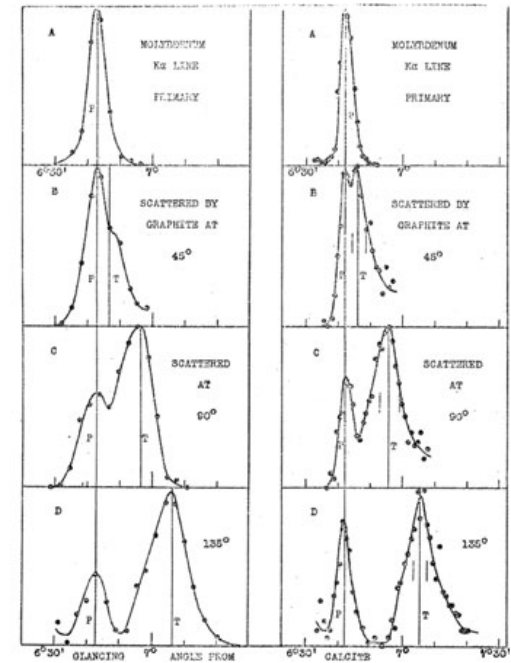
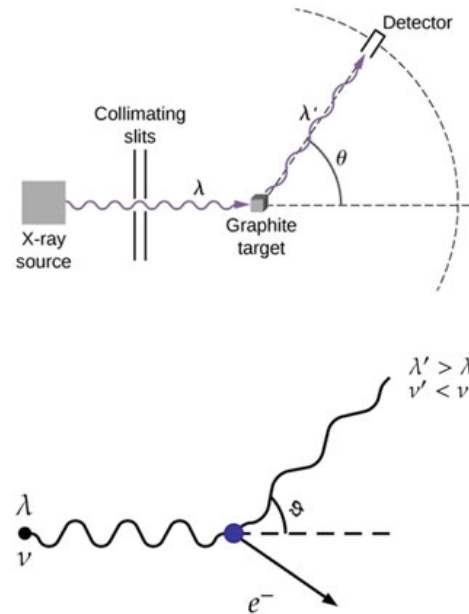
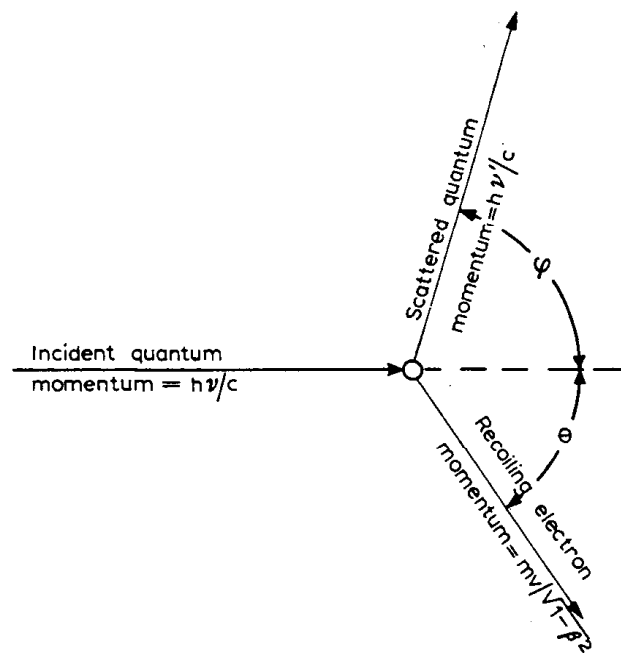
1922–1923
THE COMPTON EFFECT
AS A TURNING POINT

THE PHOTON AND ITS LINEAR MOMENTUM

ARTHUR H. COMPTON

X-rays as a branch of optics

Nobel Lecture, December 12, 1927

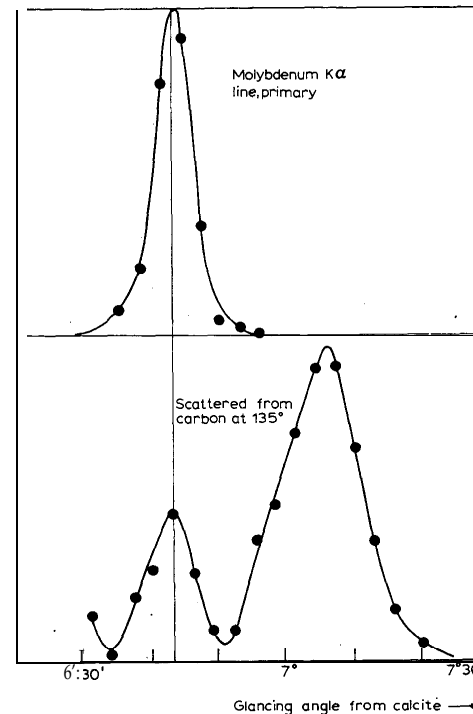


PHOTON MOMENTUM AND RELATIVISTIC SCATTERING

$$\delta\lambda = \frac{h}{mc} (1 - \cos \varphi)$$

$$E_{kin} = h\nu \cdot \frac{h\nu}{mc^2} \cos^2\theta$$

$$\cot \frac{1}{2}\varphi = \tan \theta$$



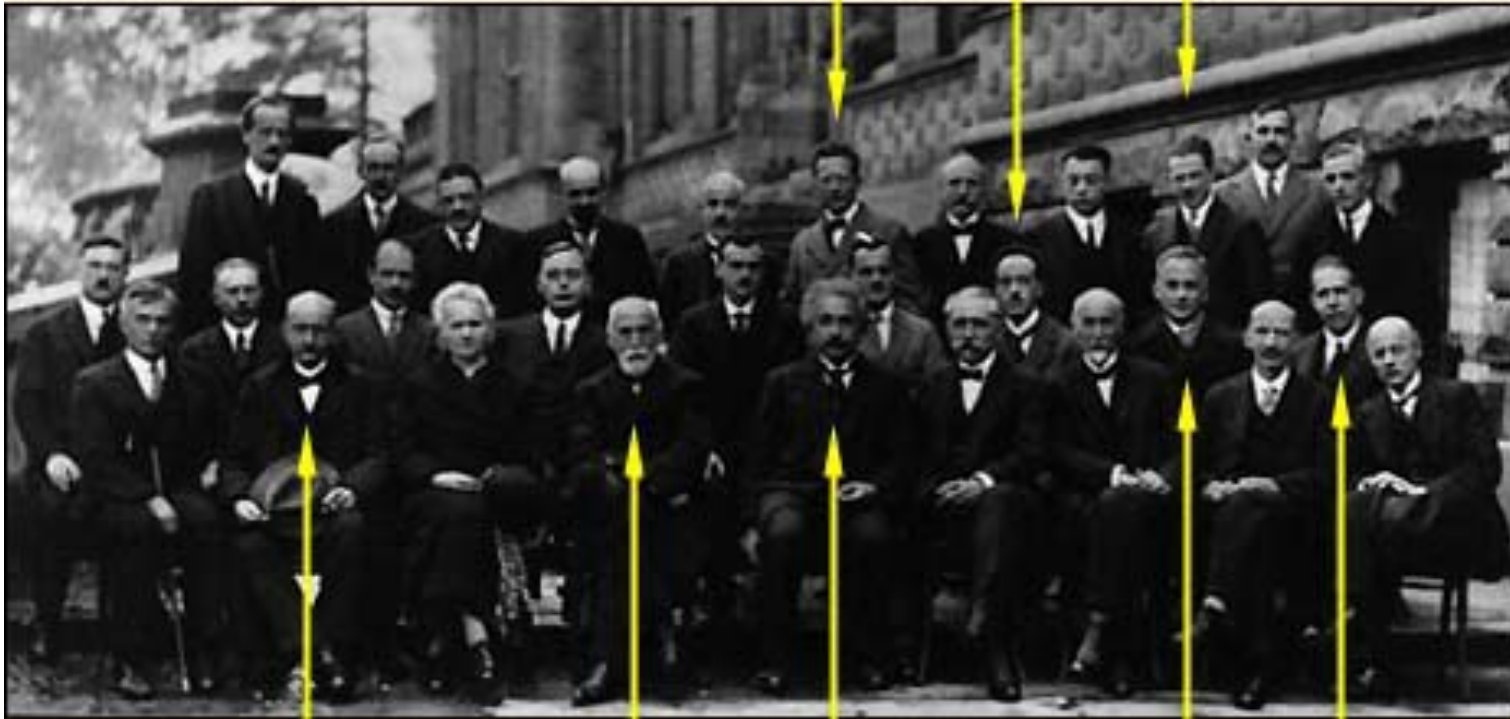
The evidence for the existence of directed quanta of radiation afforded by this experiment is very direct. The experiment shows that associated with each recoil electron there is scattered X-ray energy enough to produce a secondary ray, and that this energy proceeds in a direction determined at the moment of ejection of the recoil electron. Unless the experiment is subject to improbably large experimental errors, therefore, the scattered X-rays proceed in the form of photons.

The Solvay Congress of 1927

Werner Heisenberg

Louis de Broglie

Erwin Schrödinger



H. A. Lorentz

Max Born

Max Planck

Einstein

Niels Bohr

A LONG WAY TO GO

The half-hearted Planckian notion of energy quantization was a large step away from true quantization of the radiation field. The insight about the reality of 'light quanta' (by Einstein (1905) but still under the cautious heading "a heuristical point of view") or of "*Lichtatome*" (by Wolfke (1913, p. 1123), of "light corpuscles" or "photons" (by Gilbert Lewis (1926a) and Band (1927) or Lewis (1926b)), developed only stepwise in a gradual process along a path that was anything but straight.

**LAST LECTURE ON OCTOBER 15
TIME 16:00**

CONTEST#3

(fp./ e-mail: fulvioparmigiani@units.it)

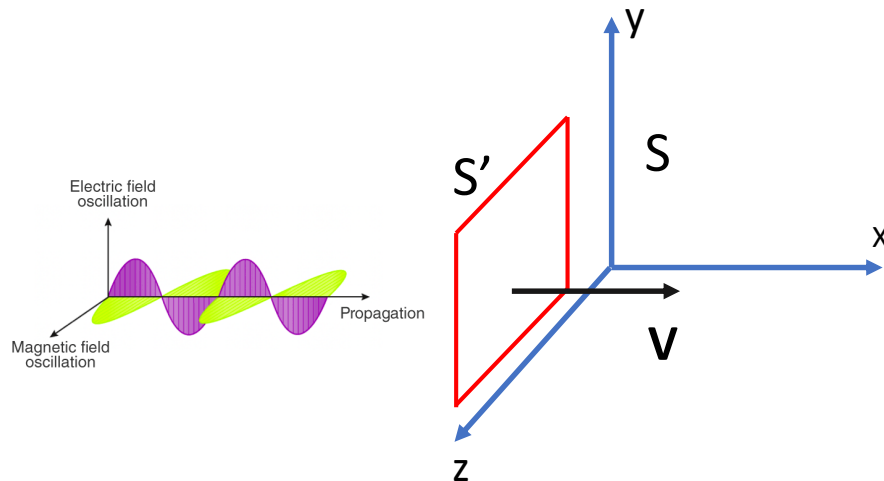
Riflessione di un'onda e.m. da uno specchio piano in movimento

Un'onda elettromagnetica di frequenza ω e ampiezza del campo elettrico E_i , polarizzata linearmente lungo l'asse y , incide perpendicolarmente su un conduttore perfetto la cui superficie giace nel piano (y,z) . Come noto un conduttore perfetto si comporta come uno specchio perfetto, cioè $\mathbf{E} = 0$ e $\mathbf{B} = 0$ all'interno del materiale ($x > 0$).

a) Valutare il campo dell'onda riflessa e il campo elettromagnetico totale.

Lo specchio è ora messo in movimento rispetto al S.d.R del laboratorio S , con una velocità costante \mathbf{v} parallela all'asse x .

- b) Calcolare le frequenze e i campi delle onde incidenti rispetto al S.d.R. S' solidale con lo specchio.
- c) Calcolare la frequenza e i campi dell'onda riflessa nel sistema S .
- d) Discutere la continuità dei campi sulla superficie mobile dello specchio.



APPENDIX 1

Emission and absorption.

How does a gas of atoms maintain the populations of its stationary states in equilibrium with a radiation field? Einstein considered atoms having two energy levels, a and b , with energies $E_b > E_a$. (For simplicity, we assume that there is only one quantum state for each energy.) The states are occupied according to the Boltzmann distribution with probabilities $P_a = C \exp(-E_a/kT)$ and $P_b = C \exp(-E_b/kT)$, where k is Boltzmann's constant, T is temperature, and C is a normalizing factor. The atoms are bathed by blackbody radiation with a yet-to-be determined spectral density $\rho(\nu, T)$, where ν is the circular frequency. He made use of the Wien displacement law, which was based on a combination of thermodynamics and electromagnetic theory: $\rho_{\text{Wien}}(\nu, T) = \nu^3 f(\nu/T)$, in which f is an unknown function. In addition, Einstein introduced the following three processes by which atoms interact with radiation.

Spontaneous emission. Einstein proposed that an excited atom in empty space will make a transition to a lower state by a process he called spontaneous emission. The probability that this takes place in time dt is $dW = A_{ba} dt$, where A_{ba} is a constant. The novelty of this proposal may not be obvious because spontaneous emission is now familiar. However, a process that appears to happen without cause could only be called novel. Furthermore, spontaneous emission describes not a radiation rate but the rate of change of probability. Thus, the language of spontaneous emission has revolutionary implications.

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Absorption. The rate at which an oscillator absorbs energy from a force field with a broad spectrum is proportional to the spectral density of the field. Thus, Einstein asserted that the probability that an atom in the lower energy state will make a transition $a \rightarrow b$ in time dt is $dW = \rho(\nu, T) B_{ab} dt$, where B_{ab} is a constant to be determined.

Stimulated emission. An oscillator absorbs energy or emits energy depending on its phase with respect to the driving force. Consequently, Einstein argued that a radiation field causes an atom in the upper energy state to make a transition to the lower state at a rate proportional to the radiation density, a process he named stimulated emission. The probability of making a transition $b \rightarrow a$ in time dt is $dW = \rho B_{ba} dt$, where B_{ba} is a constant to be determined.

By imposing the conditions for thermal equilibrium in the presence of those processes and invoking the Wien displacement law, Einstein showed that B_{ba} is proportional to A_{ba} ; that $B_{ba} = B_{ab}$; that in making a transition, the atom emits monochromatic radiation with frequency given by the Bohr condition $\nu = (E_b - E_a)/h$, where h is Planck's constant; and that for the atoms to achieve thermal equilibrium, $\rho(\nu, T) = (8\pi h \nu^3/c^3)/(\exp[h\nu/kT] - 1)$. This last equation is the Planck radiation law

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Absorption, stimulated emission and spontaneous emission are evidently introduced by analogy with the behavior of classical oscillators (Abraham-Lorentz oscillator). Applying the Boltzmann statistics to a two-quantized state system (although it is not obvious why quantized systems should obey classical statistics), Einstein introduced a fundamental character of the Quantum Physics versus the Classical Physics: the interaction are PROBABILISTIC and NOT DETERMINISTIC. Einstein justified his assumption of a spontaneous emission rate by stating, “The statistical law assumed here corresponds to the law of a radioactive reaction, and the elementary process assumed here corresponds to a relation in which only γ rays are emitted.” The “law of a radioactive reaction” is Ernest Rutherford’s law for radioactive decay: $N(t) = N(0) \exp(-t/\tau)$, where τ is a time characteristic of the atom.

At the light of the present days QED it is clear that the spontaneous emission is the only process that require the EM field quantization.

An equally daunting problem for Einstein was the lack of any theory for calculating the spontaneous emission rate. That—and a multitude of other problems on atomic structure and dynamics—would simply have to wait for the creation of a complete quantum theory.

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This transition probability is governed by the matrix element integral

$$V(t) = \sum_i \frac{q_i}{2m_i} (\bar{\mathbf{p}}_i \cdot \bar{\mathbf{A}} + \bar{\mathbf{A}} \cdot \bar{\mathbf{p}}_i) \quad V(t) = -\frac{q}{m} \bar{\mathbf{A}} \cdot \bar{\mathbf{p}} \\ = -\frac{q}{m} \left[A_0 \hat{\boldsymbol{\varepsilon}} \cdot \bar{\mathbf{p}} e^{i(\bar{\mathbf{k}} \cdot \bar{\mathbf{r}} - \omega t)} + \text{c.c.} \right]$$

from which the transition probability is derived.

$$w_{k\ell} = \frac{\pi}{2\hbar} |V_{k\ell}|^2 \left[\delta(E_k - E_\ell - \hbar\omega) + \delta(E_k - E_\ell + \hbar\omega) \right]$$

If \mathbf{A} is a classical field this relation can describe the absorption and the stimulated emission where $\mathbf{A} \neq 0$, therefore we do not need to quantize the EM field. With the spontaneous emission the photon is emitted when $\mathbf{A} = 0$. So we need to quantize the EM field since in this case the interaction Hamiltonian will be

$$H = \sum_k \hbar\omega_k (n_k + 1/2)$$

and even with $\mathbf{n}_k = \mathbf{0}$, $H_k = 1/2(\hbar\omega)$. **This is known as the zero-point energy or quantum vacuum state**

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Robert Millikan's formulation of his standpoint at the time still sounds very skeptical:

After ten years of testing and changing and learning and sometimes blundering [...] this work resulted, contrary to my own expectation, in the first direct experimental proof [...] of the exact validity [...] of the Einstein equation and the first direct photo-electric determination of Planck's h . [...] the general validity of Einstein's equation is, I think now universally concluded, and to that extent the reality of Einstein's light quanta may be considered as experimentally established. But the conception of localized light quanta out of which Einstein got his equation must still be regarded as far from being established